Vector Representation

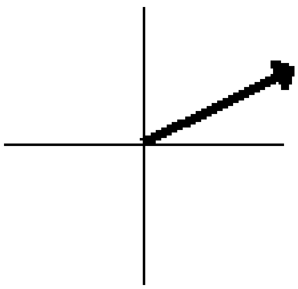
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

http://www.physicsclassroom.com/Class/vectors/u3l1a.html

MOP Connection: Vectors and Projectiles: sublevel 1

Vector quantities are quantities that have both magnitude and direction. The direction of a vector is often expressed as a counter-clockwise angle of rotation of that vector from due east (i.e., the horizontal). For questions #1-6, indicate the direction of the following vectors.

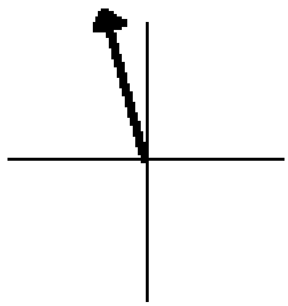
1.



CCW Dir'n: __________

magnitude: ____________

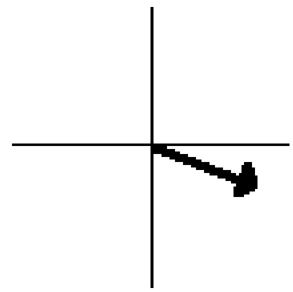
2.



CCW Dir'n: __________

magnitude: ____________

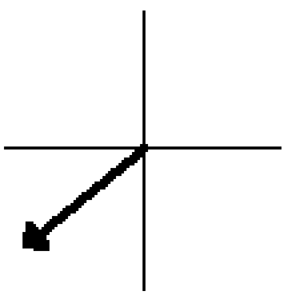
3.



CCW Dir'n: __________

magnitude: ____________

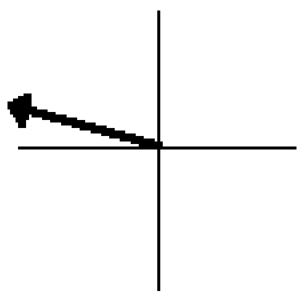
4.



CCW Dir'n: __________

magnitude: ____________

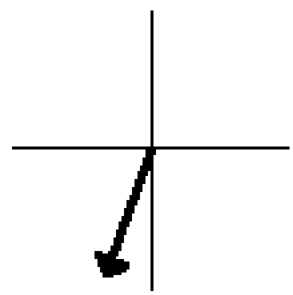
5.



CCW Dir'n: __________

magnitude: ____________

6.

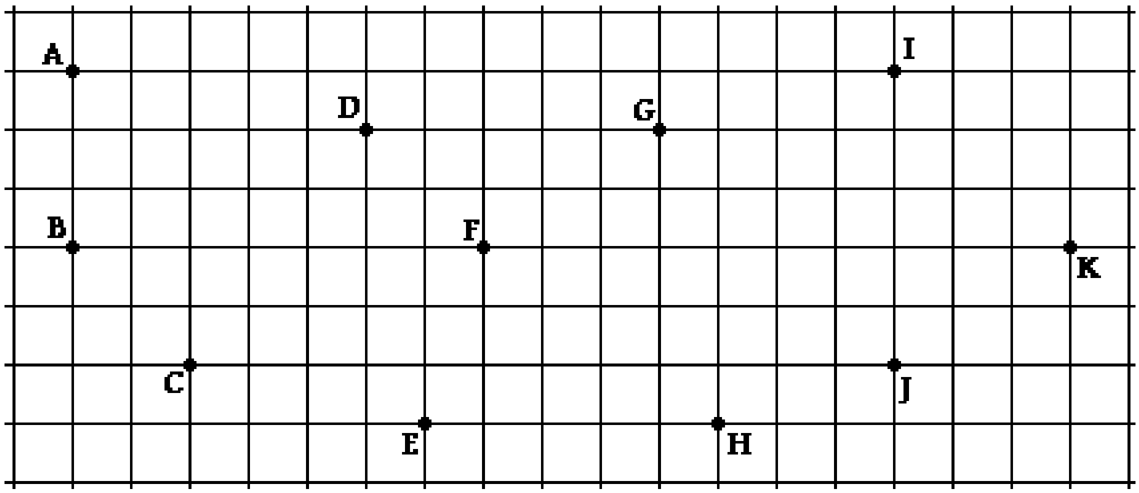


CCW Dir'n: __________

magnitude: ____________

7. The above diagrams are referred to as scaled vector diagrams. In a scaled vector diagram, the magnitude of a vector is represented by its length. A scale is used to convert the length of the arrow to the magnitude of the vector quantity. Determine the magnitude of the above six vectors if given the scale: 1 cm = 10 m/s.

8. Consider the grid below with several marked locations.



Determine the direction of the resultant displacement for a person who walks from location ...

a. A to C: ________________ b. D to B: ________________ c. G to D: ________________

d. F to A: ________________ e. F to E: ________________ f. C to H: ________________

g. E to K: ________________ h. J to K to F: ____________ i. I to K to B: ____________

9. A short verbal description of a vector quantity is given in each of the descriptions below. Read the description, select a scale, draw a set of axes, and construct a scaled vector diagram to represent the given vector quantity.

a. Kent Holditnomore excused himself from class, grabbed the cardboard pass off the lecture table, and displaced himself 10 meters at 170°.	b. Marcus Tardee took an extended lunch break and found himself hurrying through the hallways to physics class. After checking in at the attendance office, Marcus moved with an average velocity of 5.0 m/s at 305°.

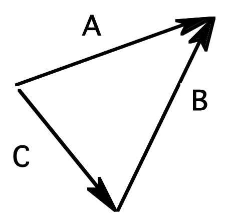
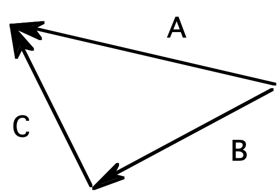
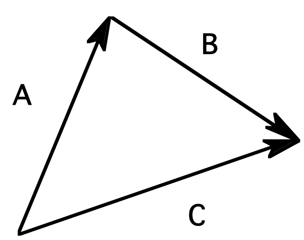
Addition of Vectors

Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

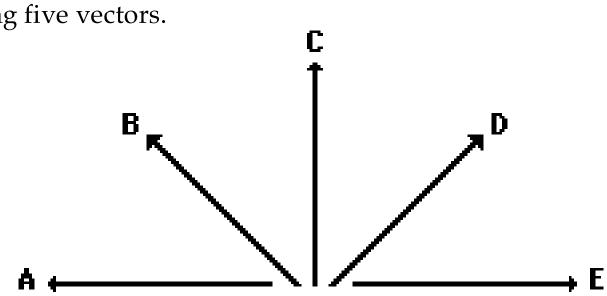
http://www.physicsclassroom.com/Class/vectors/u3l1b.html
http://www.physicsclassroom.com/Class/vectors/u3l1c.html

MOP Connection: Vectors and Projectiles: sublevels 2, 3 and 4

1. Aaron Agin recently submitted his vector addition homework. As seen below, Aaron added two vectors and drew the resultant. However, Aaron Agin failed to label the resultant on the diagram. For each case, identify the resultant (A, B, or C). Finally, indicate what two vectors Aaron added to achieve this resultant (express as an equation such as X + Y = Z) and approximate the direction of the resultant.

	Resultant is: _______ Vector Eq'n: ______________________________ Dir'n of R: ______________________________
	Resultant is: _______ Vector Eq'n: ______________________________ Dir'n of R: ______________________________
	Resultant is: _______ Vector Eq'n: ______________________________ Dir'n of R: ______________________________

2. Consider the following five vectors.



Sketch the following and draw the resultant (R). Do not draw a scaled vector diagram; merely make a sketch. Label each vector. Clearly label the resultant (R).

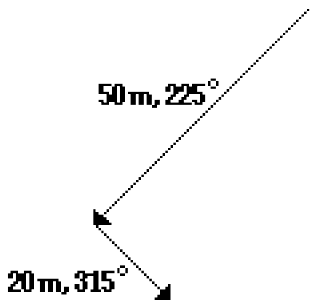
A + B + D **A + C + D** **B + C + E**

Math Skill:

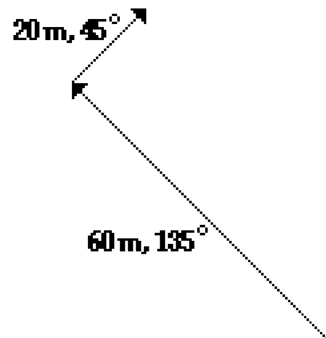
Vectors that make right angles to each other can be added together using Pythagorean theorem. Use Pythagorean theorem to solve the following problems.

3. While Dexter is on a camping trip with his Boy Scout troop, the scout leader gives each boy a compass and a map. Dexter's map contains several sets of directions. For the two sets below, draw and label the resultant (**R**). Then use the Pythagorean theorem to determine the magnitude of the resultant displacement for each set of two directions. **PSYW**

   a. Dexter walked 50 meters at a direction of 225° and then walked 20 meters at a direction of 315°.



   b. Dexter walked 60 meters at a direction of 135° and then walked 20 meters at a direction of 45°.



4. In a classroom lab, a Physics student walks through the hallways making several small displacements to result in a single overall displacement. The listings below show the individual displacements for students A and B. Simplify the collection of displacements into a pair of N-S and E-W displacements. Then use Pythagorean theorem to determine the overall displacement.

Student A	Student B
2 m, North	2 m, North
16 m, East	12 m, West
14 m, South	14 m, South
2 m, West	56 m, West
12 m, South	12 m, South
46 m, West	36 m, East
Σ E-W = ____________________	Σ E-W = ____________________
Σ N-S = ____________________	Σ N-S = ____________________
Overall Displacement:	Overall Displacement:

 Name: ______________________

Vector Components, Vector Resolution and Vector Addition

Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

http://www.physicsclassroom.com/Class/vectors/u3l1b.html
http://www.physicsclassroom.com/Class/vectors/u3l1c.html
http://www.physicsclassroom.com/Class/vectors/u3l1eb.cfm

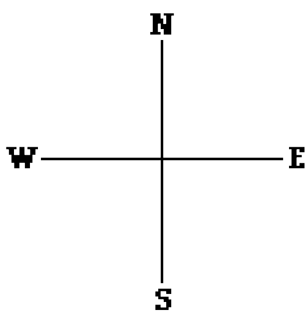
MOP Connection: Vectors and Projectiles: sublevels 3 and 5

Review: The direction of a vector is often expressed as a counter-clockwise (CCW) angle of rotation of that vector from due east (i.e., the horizontal). In such a convention, East is 0°, North is 90°, West is 180° and South is 270°.

About Vector Components:
A vector directed at 120° CCW has a direction which is a little west and a little more north. Such a vector is said to have a northward and a westward component. A **component** is simply the effect of the vector in a given direction. A hiker with a 120° displacement vector is displaced both northward and westward; there are two separate effects of such a displacement upon the hiker.

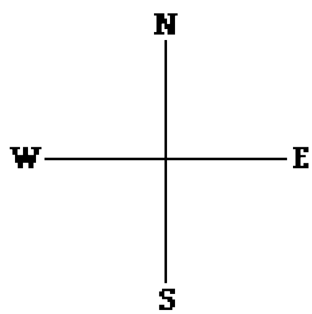
1. Sketch the given vectors; determine the direction of the two components by circling two directions (N, S, E or W). Finally indicate which component (or effect) is greatest in magnitude.

45 km, 300°



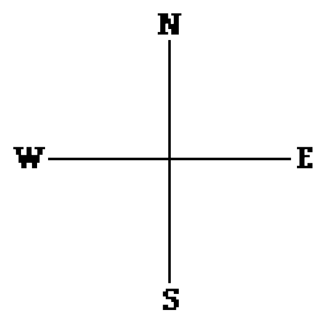
Components: **E W N S**
Greatest magnitude? ______

10 km, 265°



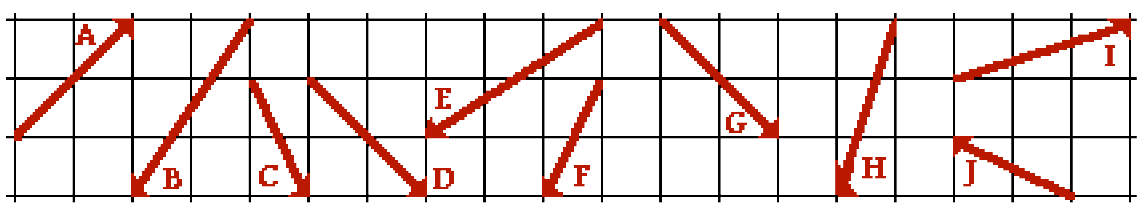
Components: **E W N S**
Greatest magnitude? ______

200 mi, 150°



Components: **E W N S**
Greatest magnitude? ______

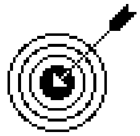
2. Consider the various vectors below. Given that each square is 10 km along its edge, determine the magnitude and direction of the components of these vectors.



Vector	E-W Component (mag .& dirn')	N-S Component (mag .& dirn')
A		
C		
E		
G		
I		

Vector	E-W Component (mag .& dirn')	N-S Component (mag .& dirn')
B		
D		
F		
H		
J		

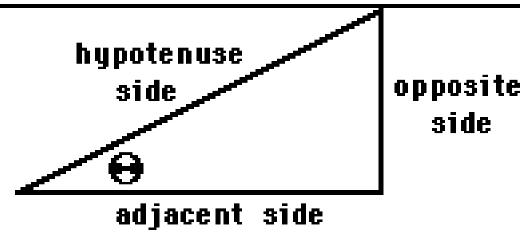
The magnitude of a vector component can be determined using trigonometric functions.



TIP
Trigonometry Review

Trigonometric functions are mathematical functions that relate the length of the sides of a right triangle to the angles of the triangle. The meaning of the functions can be easily remembered by the mnemonic

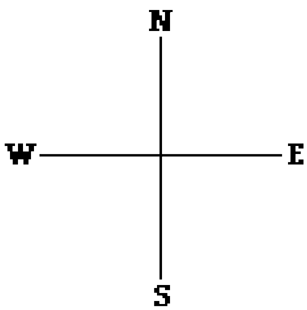
SOH CAH TOA



SOH --> $\text{Sin}\ \Theta = \dfrac{\text{Opposite}}{\text{Hypoteneuse}}$ CAH --> $\text{Cos}\ \Theta = \dfrac{\text{Adjacent}}{\text{Hypoteneuse}}$ TOA --> $\text{Tan}\ \Theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

3. Sketch the given vectors; project the vector onto the coordinate axes and sketch the components. Then determine the magnitude of the components using SOH CAH TOA.

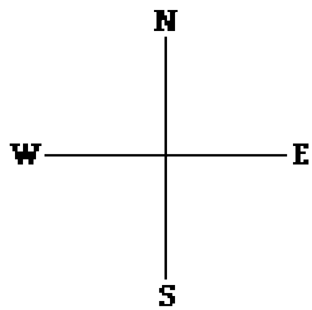
45 km, 300°



E-W Component:

N-S Component:

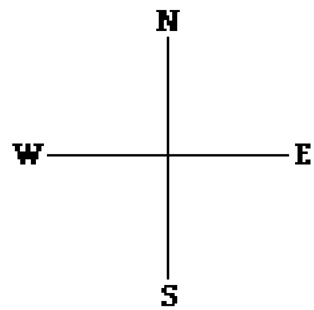
10 km, 265°



E-W Component:

N-S Component:

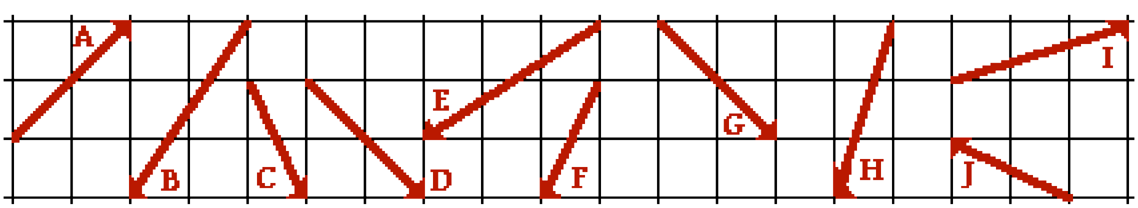
200 mi, 150°



E-W Component:

N-S Component:

4. Consider the diagram below (again); each square is 10 km along its edge. Use components and vector addition to determine the resultant displacement (magnitude only) of the following:



A + B + C ==> Σ E-W: __________ Σ N-S: ___________ Overall Displacement: _____________

D + E + F ==> Σ E-W: __________ Σ N-S: ___________ Overall Displacement: _____________

G + H + I ==> Σ E-W: __________ Σ N-S: ___________ Overall Displacement: _____________

A + J + G ==> Σ E-W: __________ Σ N-S: ___________ Overall Displacement: _____________

Vector Addition by Components

Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

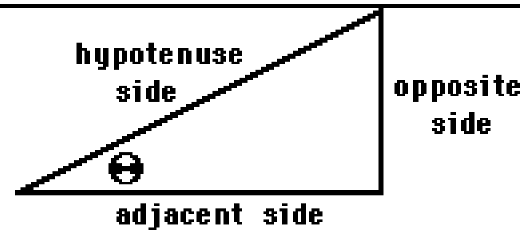
http://www.physicsclassroom.com/Class/vectors/u3l1eb.cfm

MOP Connection: Vectors and Projectiles: sublevels 3 and 4

TIP
Trigonometry Review

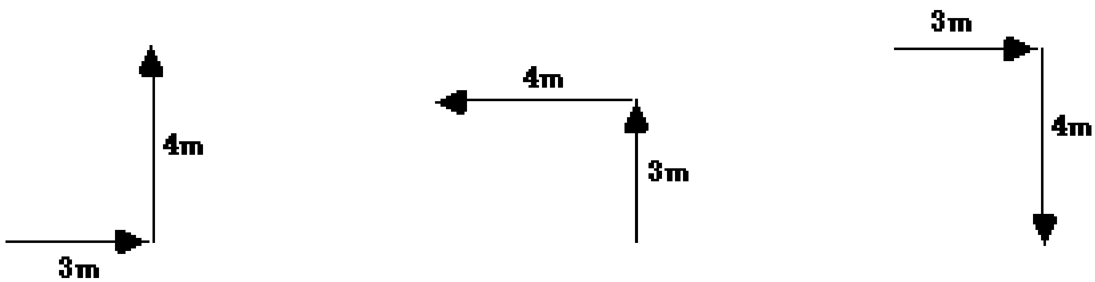
Trigonometric functions are mathematical functions that relate the length of the sides of a right triangle to the angles of the triangle. The meaning of the functions can be easily remembered by the mnemonic

SOH CAH TOA

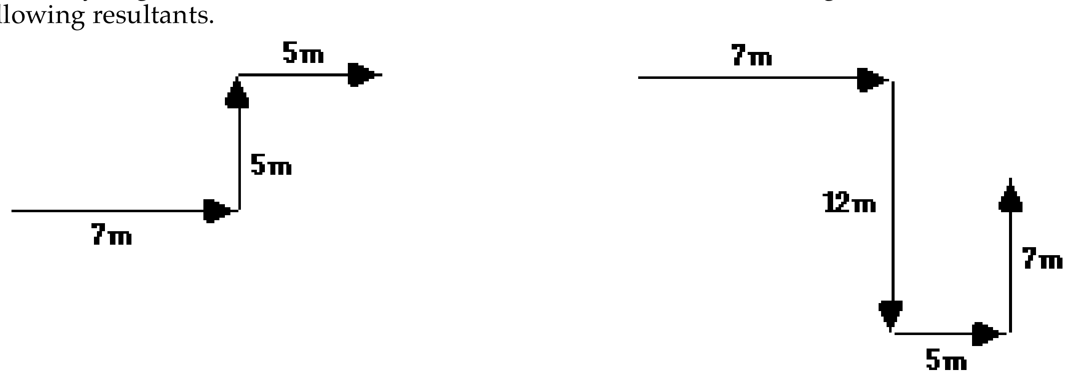


SOH --> **S**in $\Theta = \frac{\text{Opposite}}{\text{Hypoteneuse}}$ **CAH** --> **C**os $\Theta = \frac{\text{Adjacent}}{\text{Hypoteneuse}}$ **TOA** --> **T**an $\Theta = \frac{\text{Opposite}}{\text{Adjacent}}$

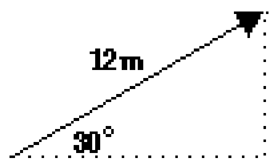
1. For the following vector addition diagrams, use Pythagorean Theorem to determine the magnitude of the resultant. Use SOH CAH TOA to determine the direction. **PSAYW**

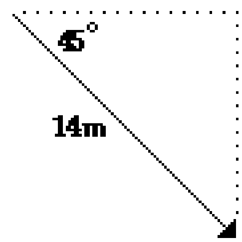


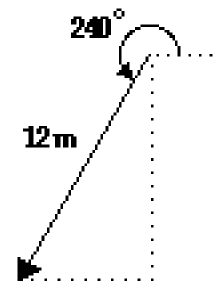
2. Use the Pythagorean Theorem and SOH CAH TOA to determine the magnitude and direction of the following resultants.



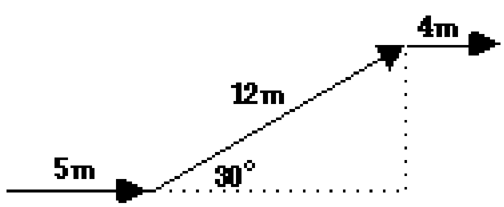
3. A component is the effect of a vector in a given x- or y- direction. A component can be thought of as the projection of a vector onto the nearest x- or y-axis. SOH CAH TOA allows a student to determine a component from the magnitude and direction of a vector. Determine the components of the following vectors.

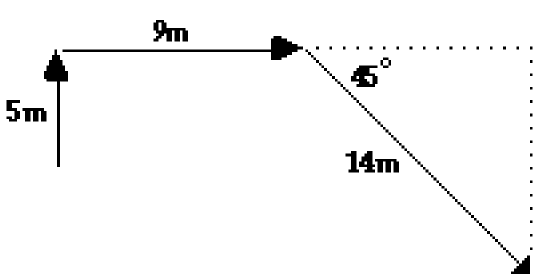






4. Consider the following vector diagrams for the displacement of a hiker. For any *angled* vector, use SOH CAH TOA to determine the components. Then sketch the resultant and determine the magnitude and direction of the resultant.





Relative Velocity and Riverboat Problems

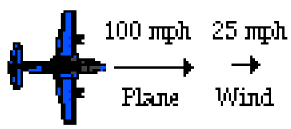
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

http://www.physicsclassroom.com/Class/vectors/u3l1f.html
http://www.physicsclassroom.com/Class/vectors/u3l1g.html

MOP Connection: Vectors and Projectiles: sublevel 6 (and maybe sublevel 5)

1. Planes fly in a medium of moving air (winds), providing an example of relative motion. If the speedometer reads 100 mi/hr, then the plane moves 100 mi/hr relative to the air. But since the air is moving, the plane's speed relative to the ground will be different than 100 mi/hr. Suppose a plane with a 100 mi/hr air speed encounters a tail wind, a head wind and a side wind. Determine the resulting velocity (magnitude and CCW direction) of the plane for each situation.

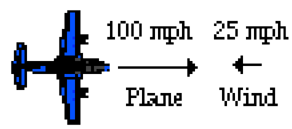
Tail Wind



Magnitude:

CCW Direction:

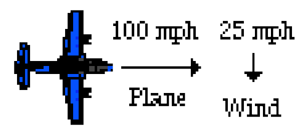
Head Wind



Magnitude:

CCW Direction:

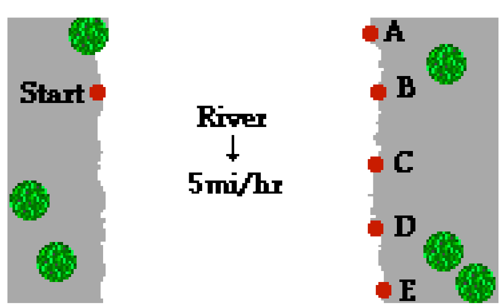
Side Wind



Magnitude:

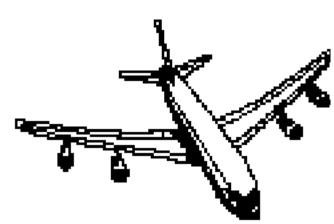
CCW Direction:

2. The situation of a plane moving in the medium of moving air is similar to a motorboat moving in the medium of moving water. In a river, a boat moves relative to the water and the water moves relative to the shore. The result is that the resultant velocity of the boat is different than the boat's speedometer reading, *thanks to* the movement of the water that the boat is in. In the diagram below, a top view of a river is shown. A boat starts on the west side (left side) of the river and heads a variety of directions to get to the other side. The river flows south (down). Match the boat headings and boat speeds to the indicated destinations. Use each letter once.



Boat Heading	Boat Speed	Destination (A, B, C, D or E)
→	14mi/hr	
→	7mi/hr	
↗	7mi/hr	
→	20mi/hr	
↗	12mi/hr	

3. A pilot wishes to fly due North from the Benthere Airport to the Donthat Airport. The wind is blowing *out of the Southwest* at 30 mi/hr. The small plane averages a velocity of 180 mi/hr. What heading should the pilot take? Use a sketch to help solve.



4. A riverboat heads east on a river that flows north. The riverboat is moving at 5.1 m/s with respect to the water. The water moves north with respect to the shore at a speed of 3.6 m/s.
   a. Determine the resultant velocity of the riverboat (velocity with respect to the shore).

   b. If the river is 71.0 m wide, then determine the time required for the boat to cross the river.

   c. Determine the distance that the boat will travel downstream.

5. Suppose that the boat attempts this same task of crossing the river (5.1 m/s with respect to the water) on a day in which the river current is greater, moving at 4.7 m/s with respect to the shore. Determine the same three quantities - (a) resultant velocity, (b) time to cross the river, and (c) distance downstream.

6. For a boat *heading* straight across a river, does the speed at which the river flows effect the time required for the boat to cross the river? ________ Explain your answer.

7. Repeat the same three riverboat calculations for the following two sets of given quantities.

<table>
<tr>
<td>Velocity of boat (w.r.t. water) = 3.2 m/s, East
Velocity of river (w.r.t. shore) = 4.4 m/s, South
Width of river = 127 m

a. Resultant velocity:
magnitude = ______________________
direction = ________________________

b. Time to cross river = _______________

c. Distance downstream = ____________</td>
<td>Velocity of boat (w.r.t. water) = 2.6 m/s, West
Velocity of river (w.r.t. shore) = 4.2 m/s, South
Width of river = 96 m

a. Resultant velocity:
magnitude = ______________________
direction = ________________________

b. Time to cross river = _______________

c. Distance downstream = ____________</td>
</tr>
</table>

Projectile Motion

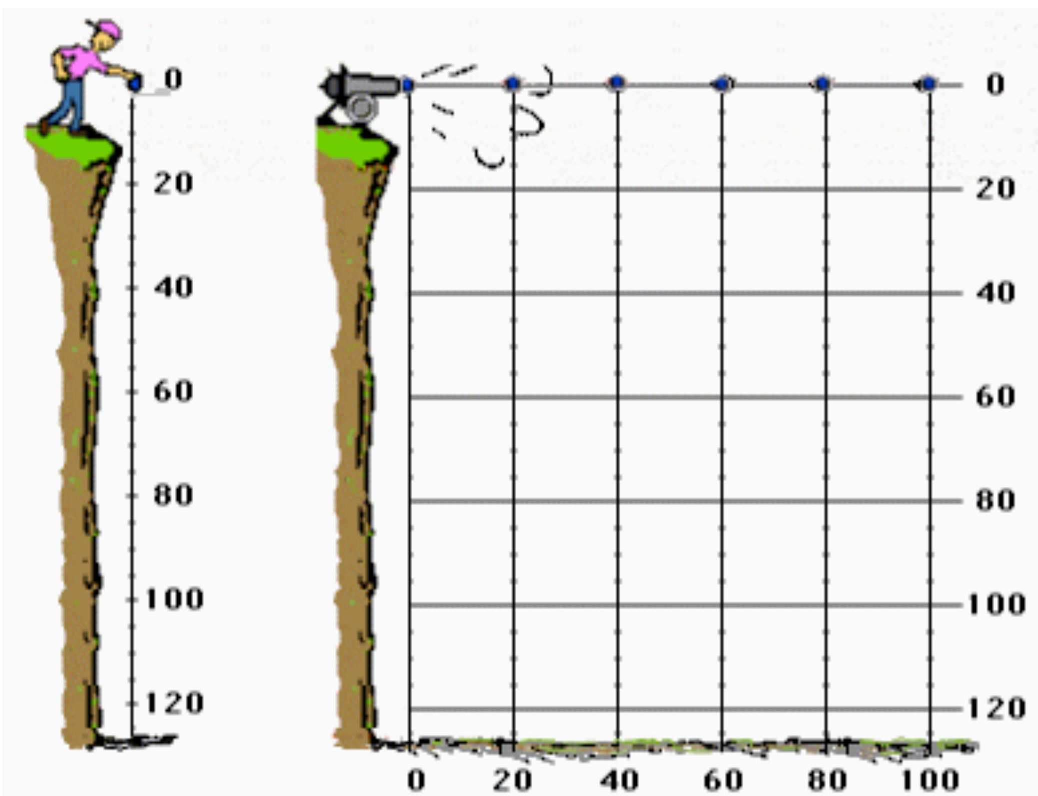
Read from **Lesson 2** of the **Vectors and Motion in Two-Dimensions chapter** at **The Physics Classroom**:

http://www.physicsclassroom.com/Class/vectors/u3l2a.html
http://www.physicsclassroom.com/Class/vectors/u3l2b.html
http://www.physicsclassroom.com/Class/vectors/u3l2c.html
http://www.physicsclassroom.com/Class/vectors/u3l2d.html
http://www.physicsclassroom.com/Class/vectors/u3l2e.html
http://www.physicsclassroom.com/Class/vectors/u3l2f.html

MOP Connection: Vectors and Projectiles: sublevels 7 - 10

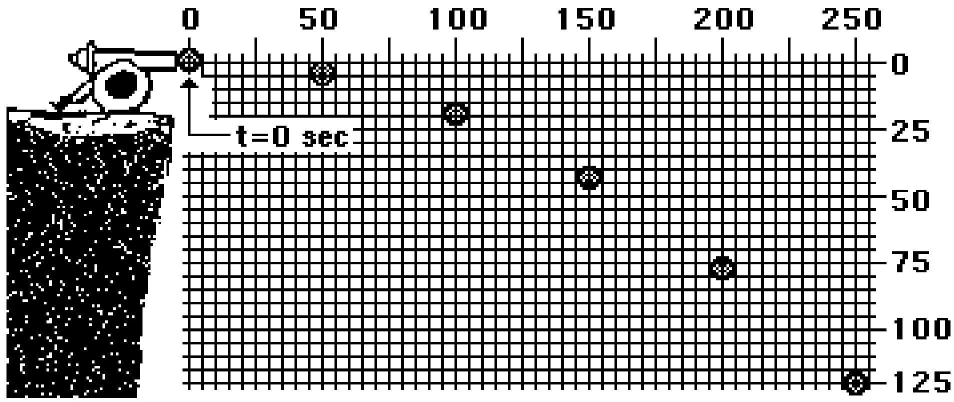
1. A baseball is dropped off a cliff and it accelerates to the ground at a rate of -9.8 m/s^2, down. Meanwhile a cannonball is launched horizontally from a cannon with a horizontal speed of 20 m/s.
2. A scale is shown along the sides of the graphic at the right. Use the scale to locate the position of the baseball and the cannonball. Trace a line to indicate the trajectory of the cannonball.

Baseball		**Cannonball**		
t(s)	**y(m)**	**t(s)**	**x(m)**	**y(m)**
0				
1				
2				
3				
4				
5				



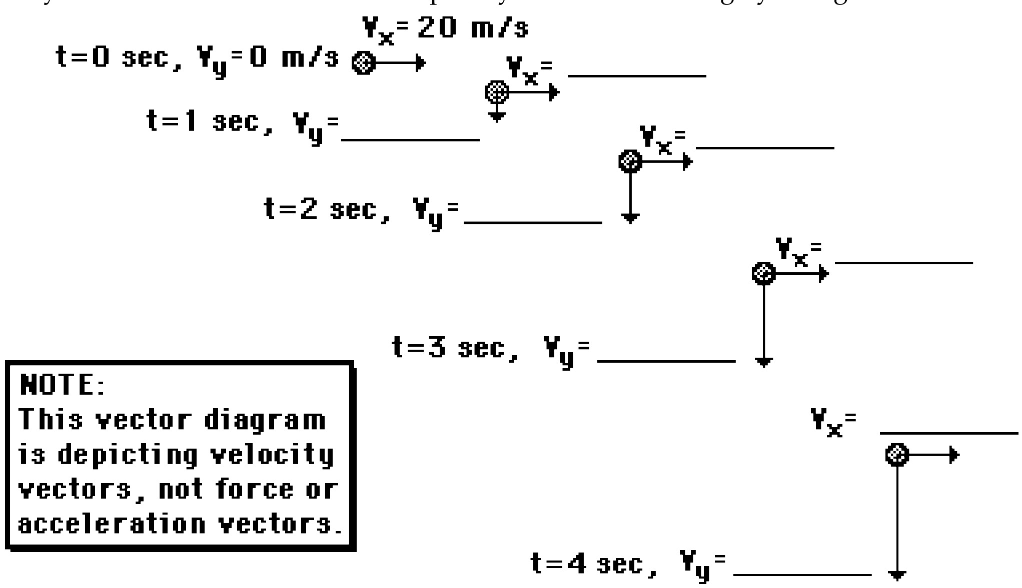
3. Which of these two balls strike the ground first? ________
4. Compare the two diagrams - the vertical free-fall motion on the left and the two-dimensional free-fall motion on the right. Describe the effect on an object's horizontal motion upon the object's vertical motion.

5. The diagram below shows the trajectory of a horizontally launched projectile. Positions of the projectile at 1-second intervals are shown. Demonstrate your understanding of the components of the displacement vector by determining the horizontal displacement (x) and the vertical displacement (y) after the fifth second.



x = ______________________ y = ________________________

6. A ball is launched horizontally from the top of a cliff with an initial velocity of 20 m/s. The trajectory of the ball is shown below. Express your understanding by filling in the blanks.

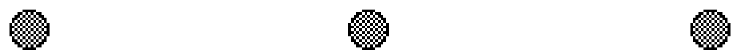


7. If the ball in the diagram above strikes the ground after four seconds, then (a) how high was the cliff and (b) how far from the base of the cliff will the ball land? **PSYW**

8. If the ball's initial speed in question #6 was 16 m/s, then how far from the cliff will the ball land?

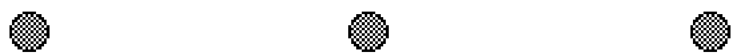
9. Use the diagram below to construct a free-body diagram for a vertically launched projectile as it rises towards its peak, at its peak, and as it is falls from its peak.

Before Peak: **At Peak:** **After Peak:**



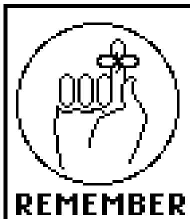
10. Use the diagram below to construct a free-body diagram for a projectile launched at an angle as it rises towards its peak, at its peak, and as it is falls from its peak.

Before Peak: **At Peak:** **After Peak:**



11. A projectile is launched with a speed of 31.1 m/s at an angle of 71.2 degrees above the horizontal. The horizontal and vertical components of the initial velocity are shown in the first row of the data table. Fill in the table indicating the value of the horizontal and vertical components of velocity for the projectile during the course of its motion.

Time (s)	v_x (m/s)	v_y (m/s)	a_x (m/s^2)	a_y (m/s^2)
0	10.0	29.4		
1				
2				
3				
4				
5				
6				



Key Concepts:
A projectile is an object that has the following characteristics.
- The only force acting on it is a gravitational force; it is a free-falling object.
- The acceleration is directed downwards and has a value of 9.8 m/s^2.
- Once projected, it continues its horizontal motion without any need of a force.
- As it rises, its vertical velocity (v_y) decreases; as it falls, its v_y increases.
- As it travels through the air, its horizontal velocity remains constant.

The Equations:
Kinematic equations used for 1-dimensional motion can be used for projectile motion as well. The key to their use is to remember that perpendicular components of motion are independent of each other. As such, the equations for one dimension must be applied to either the horizontal motion of a projectile or the vertical motion of a projectile. When using the equations to analyze projectile motion, one assumes negligible air resistance and an acceleration of gravity of 9.8 m/s^2, down(-). Thus, $a_x = 0$ m/s/s and $a_y = -9.8$ m/s/s.

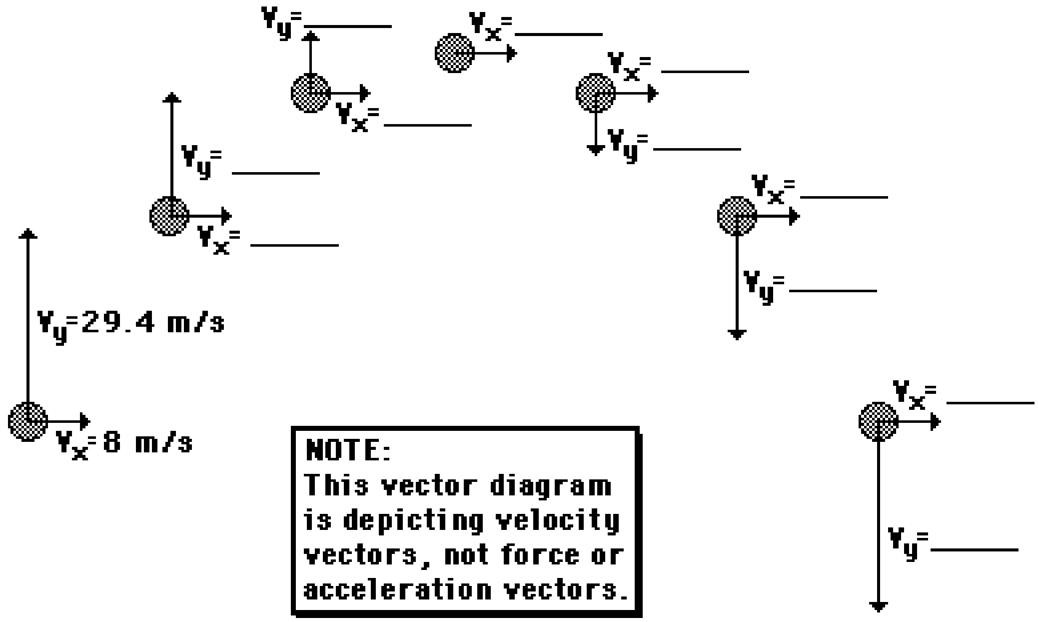
1-Dim.	$v_f = v_o + a \bullet t$	$d = v_o \bullet t + \frac{1}{2} \bullet a \bullet t^2$	$v_f^2 = v_o^2 + 2 \bullet a \bullet d$	$d = \frac{v_o + v_f}{2} \bullet t$
x-comp.	$v_{fx} = v_{ox} + a_x \bullet t$	$d_x = v_{ox} \bullet t + \frac{1}{2} \bullet a_x \bullet t^2$	$v_{fx}^2 = v_{ox}^2 + 2 \bullet a_x \bullet d_x$	$d_x = \frac{v_{ox} + v_{fx}}{2} \bullet t$
y-comp.	$v_{fy} = v_{oy} + a_y \bullet t$	$d_y = v_{oy} \bullet t + \frac{1}{2} \bullet a_y \bullet t^2$	$v_{fy}^2 = v_{oy}^2 + 2 \bullet a_y \bullet d_y$	$d_y = \frac{v_{oy} + v_{fy}}{2} \bullet t$

12. A ball is projected horizontally from the top of a 92.0-meter high cliff with an initial speed of 19.8 m/s. Determine: (a) the horizontal displacement, and (b) the final speed the instant prior to hitting the ground.

13. Determine the launch speed of a horizontally launched projectile that lands 26.3 meters from the base of a 19.3-meter high cliff.

14. A soccer ball is kicked horizontally at 15.8 m/s off the top of a field house and lands 33.9 metes from the base of the field house. Determine the height of the field house.

15. A ball is projected at an angle with an initial horizontal velocity of 8.0 m/s and an initial vertical velocity of 29.4 m/s. The trajectory diagram shows the position of the ball after each consecutive second. Express your understanding of projectiles by filling in the blanks.



16. Determine … (a) … the displacement of the ball, (b) … the height above the ground at its peak, and (c) … the final speed of the ball upon hitting the ground.

17. Suppose that the horizontal component of the initial velocity had been 13.0 m/s and the vertical velocity had been unchanged (in questions #15 and #16). Determine the … (a) … time of flight, (b) … the displacement of the ball, (c) … the height above the ground, and (d) … the speed upon hitting the ground.

18. A physics student is driving his pick-up truck down Lake Avenue. The pick-up is equipped with a projectile launcher that imparts a vertical velocity to a water-filled rubber projectile. While traveling 20.0 m/s in an eastward direction, the projectile is launched vertically with a velocity of 58.8 m/s.

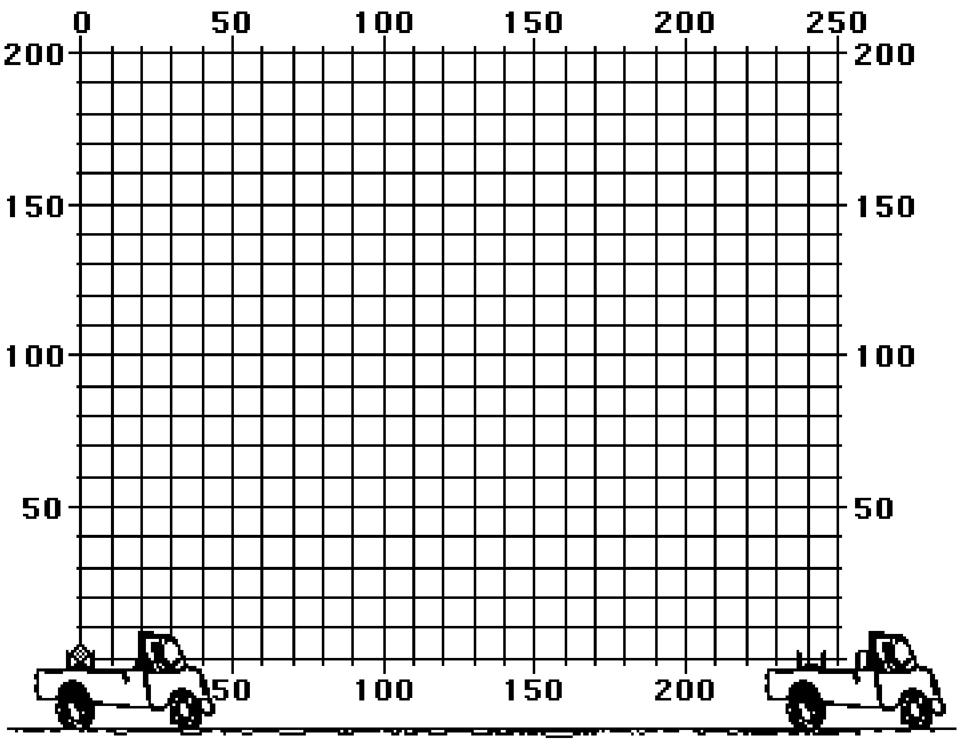
    Fill in the table at the right. showing the horizontal and vertical displacement of the projectile every second for the first 12 seconds.

    $$d_x = v_{ox} \bullet t + \frac{1}{2} \bullet a_x \bullet t^2$$

    $$d_y = v_{oy} \bullet t + \frac{1}{2} \bullet a_y \bullet t^2$$

t (s)	d_x **(m)**	d_y **(m)**
0.0		
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		
11.0		
12.0		

19. On the diagram below, place a large dot on the location of the projectile during each second of its trajectory. Draw a smooth curve through the dots to indicate the trajectory.



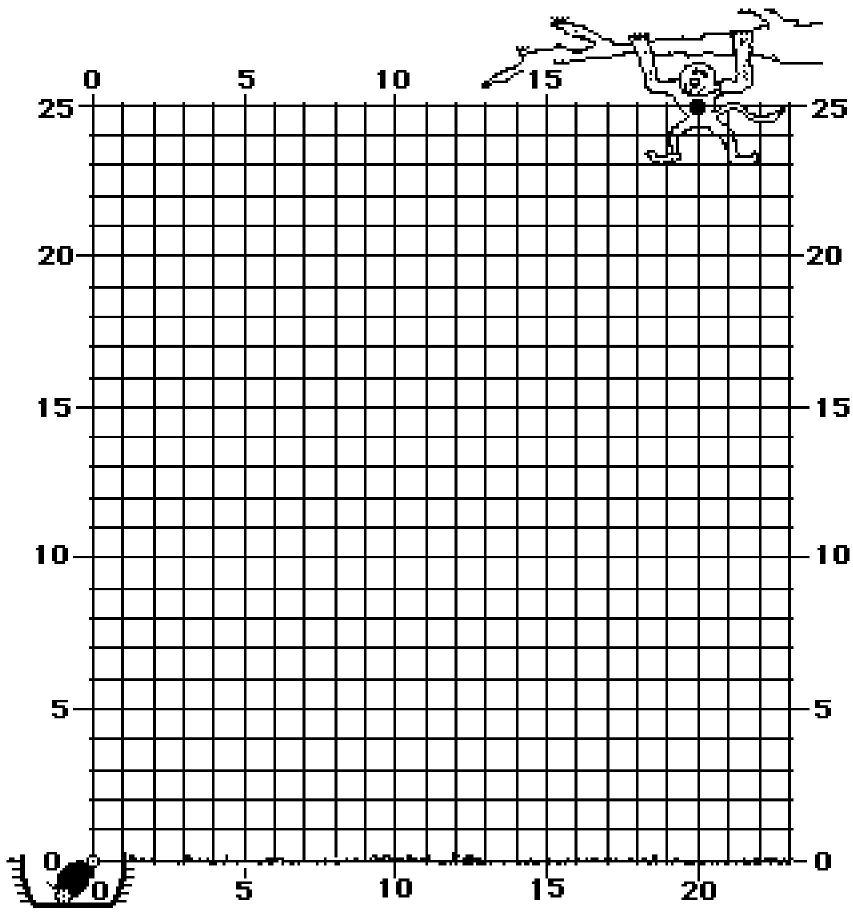
20. Will the projectile land in the truck, behind the truck or in front of the truck? (Assume no air resistance.) ________________ Explain your answer.

21. A zookeeper has a monkey that he must feed daily. The monkey spends most of the day in the trees just hanging from a branch. When the zookeeper launches a banana to the monkey, the monkey has the peculiar habit of dropping from the trees the moment that the banana is launched.

    The banana is launched with a speed of 16.0 m/s at a direction of 51.3° above the horizontal (which would be directly at the monkey). The monkey is initially at rest in a tree 25.0-m above the ground. Use kinematic equations to determine the horizontal and vertical displacements of the banana and the monkey at 0.5-second time intervals. Then plot the trajectories of both banana and monkey on the diagram below.

Banana			
Time (s)	**d_x (m)**	**d_y (m)**	**Height (m)**
0	0	0	0
0.5			
1.0			
1.5			
2.0			

Monkey		
Time (s)	**d_y (m)**	**Height (m)**
0	0	25.0
0.5		
1.0		
1.5		
2.0		



22. Based on your mathematical analysis above, will the zookeeper hit the monkey if she aims the banana directly at the monkey? ___________

23. Use trigonometric functions to resolve the following velocity vectors into horizontal and vertical components. Then utilize kinematic equations to calculate the other motion parameters. Be careful with the equations; be guided by the principle that "perpendicular components of motion are independent of each other."

A long jumper leaps with an initial velocity of 9.5 m/s at an angle of 40° to the horizontal.	Megan Progress, GBS golf standout, hits a nine-iron with a velocity of 25 m/s at an angle of 60° to the horizontal.	A place kicker launches a kickoff at an angle of 30° to the horizontal and a velocity of 30 m/s.
v_{ox} = ____________ m/s	v_{ox} = ____________ m/s	v_{ox} = ____________ m/s
v_{oy} = ____________ m/s	v_{oy} = ____________ m/s	v_{oy} = ____________ m/s
t_{up} = ____________ s	t_{up} = ____________ s	t_{up} = ____________ s
t_{total} = ____________ s	t_{total} = ____________ s	t_{total} = ____________ s
d_x = ____________ m	d_x = ____________ m	d_x = ____________ m
d_y @ peak = ________ m	d_y @ peak = ________ m	d_y @ peak = ________ m

24. Generalize the calculations performed in question #23 above by writing the equations used to calculate each of the quantities requested in the problem.

v_{ox} = ______________________ v_{oy} = ______________________

t_{up} = ______________________ t_{total} = ______________________

d_x = ______________________ d_y @ peak = ______________________

25. Determine the range of a ball launched with a speed of 40.0 m/s at angles of (a) 40.0 degrees, (b) 45.0 degrees, and (c) 50.0 degrees from ground level. **PSYW** and label your answers.

26. For the three initial launch angles in question #25, determine the peak heights. **PSYW** and label your answers.