

The Harmonics of Vibrating Strings

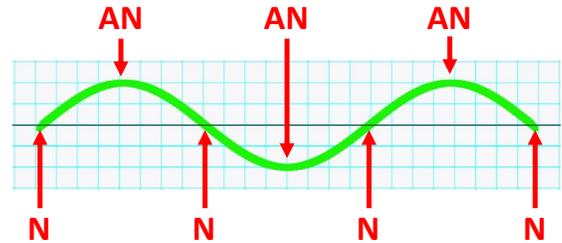
Video Notes

Standing Wave Patterns

- Each frequency at which a string naturally vibrates is associated with a standing wave pattern.
- Standing wave patterns consist of **nodes** and **antinodes**.

A **node** is a location along the string that appears to be standing still. It is a point of "no-des"-placement.

An **antinode** is the opposite – a point that is vibrating wildly from a maximum positive displacement to a maximum negative displacement.



Harmonic Frequencies and Their Relationships

- One of the frequencies at which a string **naturally** vibrates.
- The lowest-frequency harmonic is the first harmonic or fundamental frequency.
- Other frequency values are whole number multiples of the fundamental frequency

$$f_1 = 150 \text{ Hz}$$

$$f_2 = 300 \text{ Hz}$$

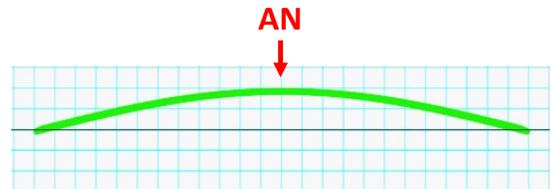
$$f_3 = 450 \text{ Hz}$$

$$f_4 = 600 \text{ Hz}$$

Just an example.

Length and Wavelength Relationships

- The standing wave pattern for the first harmonic has one anti-node and one-half a wavelength in the string.
- The wavelength of the first harmonic is ALWAYS twice the string length.
- Other harmonics have wavelengths that are fractions of this wavelength.



If $L = 60 \text{ cm}$, then $\lambda_1 = 120 \text{ cm}$.

$$\lambda_2 = (1/2) \cdot \lambda_1$$

$$\lambda_2 = 60 \text{ cm}$$

$$\lambda_3 = (1/3) \cdot \lambda_1$$

$$\lambda_3 = 40 \text{ cm}$$

$$\lambda_4 = (1/4) \cdot \lambda_1$$

$$\lambda_4 = 30 \text{ cm}$$

Comparing Patterns, Frequencies and Wavelengths for Other Harmonics

Second Harmonic:

Two antinodes in the pattern

$$f_2 = 2 \cdot f_1$$

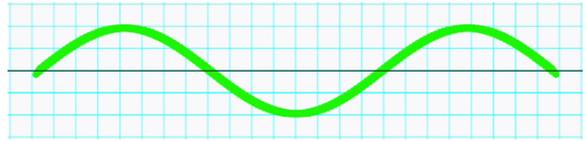
$$\lambda_2 = \frac{1}{2} \cdot \lambda_1$$



Third Harmonic:

Three antinodes in the pattern

$$f_3 = 3 \cdot f_1 \quad \lambda_3 = \frac{1}{3} \cdot \lambda_1$$

**Fourth Harmonic:**

Four antinodes in the pattern

$$f_4 = 4 \cdot f_1 \quad \lambda_4 = \frac{1}{4} \cdot \lambda_1$$



| Harmonic | Pattern | # of ANs | λ | f | Examples | |
|-----------------|---------|----------|---------------|---------------|---------------|----------------|
| | | | | | f (Hz) | λ (cm) |
| 1 st | | 1 | λ_1 | f_1 | 150 | 120 |
| 2 nd | | 2 | $\lambda_1/2$ | $2 \cdot f_1$ | 300 | 60 |
| 3 rd | | 3 | $\lambda_1/3$ | $3 \cdot f_1$ | 450 | 40 |
| 4 th | | 4 | $\lambda_1/4$ | $4 \cdot f_1$ | 600 | 30 |
| 5 th | | 5 | $\lambda_1/5$ | $5 \cdot f_1$ | 750 | 24 |
| n th | -- | n | λ_1/n | $n \cdot f_1$ | $150 \cdot n$ | $1.20/n$ |

Follow along with the video and show the solution for the following two examples.

Example 1

A vibrating string has a first harmonic of 100 Hz. What would the standing wave pattern look like for the same string when vibrating at 500 Hz?

Example 2

A string is 75 cm long. What would the standing wave pattern look like for a standing wave with a wavelength of 50 cm?