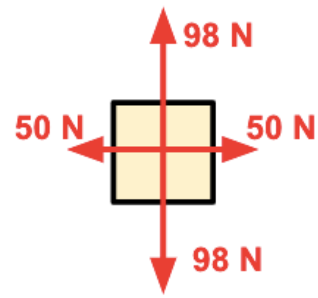
Equilibrium
Lesson Notes

Learning Outcomes
- What is meant by saying "an object is at equilibrium"?
- How do you mathematically analyze the forces on an object that is at equilibrium?

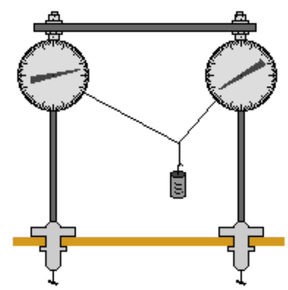
What is Equilibrium?
Equilibrium is the state of an object upon which all the individual forces are balanced. Objects at equilibrium will have the following traits.

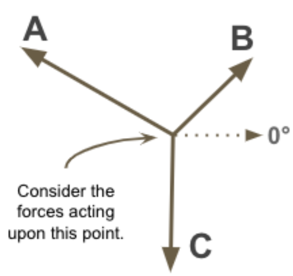
- Individual forces are balanced.
- The net force (F_{net}) is 0 N
- The acceleration is 0 m/s/s.
- The velocity is constant (but not necessarily 0).

NOTE: For an object at equilibrium, the forces are balanced but not necessarily equal to each other.



Example 1: An Equilibrium "Lab"
A 1.0-kg mass is hung by two strings. Force scales are used to measure the force in the strings. Analyze the forces to show that the object is at equilibrium.





Force	Magnitude	Direction
A	3.4 N	161° CCW
B	9.2 N	70° CCW
C	9.8 N	270°CCW

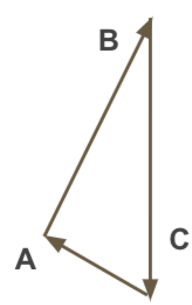
Graphical Analysis - Scaled Vector Diagram
The net force is the vector sum of all the forces (∑F). So if the three forces are added as vectors, they would add up to 0 N. Their **resultant** would be 0 N.

Vector A
3.4 N
161° CCW

Vector B
9.2 N
70° CCW

Vector C
9.8 N
270° CCW

Scaled Vector Addition Diagram



Finding:
The head of C ends at the tail of A. The resultant is 0.

Component Method of Analysis

When adding angled vectors, it is a common strategy to determine the components of each vector … and to add the components.

Vector	x-Component	y-Component
A **3.4 N, 161°**	A_x = 3.4 N • cos(161°) A_x = 3.2 N, left	A_x = 3.4 N • sin(161°) A_x = 1.1 N, up
B **9.2 N, 70°**	B_x = 9.2 N • cos(70°) B_x = 3.1 N, right	B_x = 9.2 N • sin(70°) B_x = 8.6 N, up
C **9.8 N, 270°**	C_x = 9.8 N • cos(270°) C_x = 0 N	C_x = 9.8 N • sin(270°) C_x = 9.8 N, down
Sum	0.1 N, left **(~0 N)**	0.1 N, down **(~0 N)**

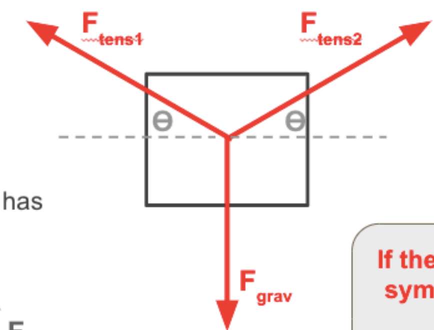
Conclusion

Within the precision of our measuring tools, the results show that ∑F = 0 N.

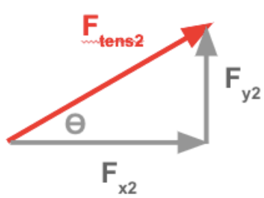
Sign Hanging

The hanging of a sign or other massive object by ropes, wires, or cables demonstrates the importance of an equilibrium analysis.

Three Forces:
Tension (up, left)
Tension (up, right)
Gravity (down)



Each tension force has two components



$F_y = F_{tens} \cdot \sin ϴ$

If all the forces are balanced, then …

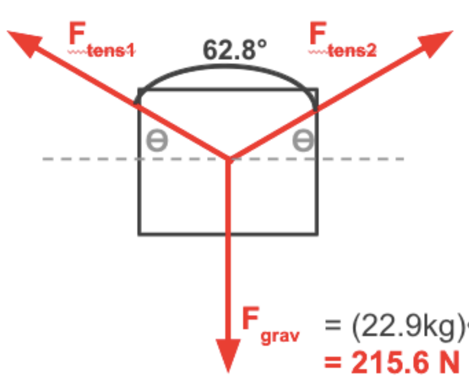
- **$F_{x1} = F_{x2}$**
- **$F_{grav} = F_{y1} + F_{y2}$**

If the sign is hung symmetrically …

$F_{y1} = F_{y2} = ½ \cdot F_{grav}$

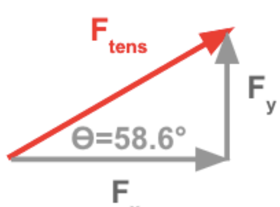
A Sign Hanging Example

A 22.9-kg sign is supported by two cables that make an angle of 62.8° with one another. Determine the tension in each cable.



F_{grav} = (22.9kg)•(9.8 N/kg)
= 215.6 N

2•ϴ + 62.8 = 180.0
ϴ = 58.6°



$F_y = ½ \cdot F_{grav}$
F_y = 107.8 N

$F_y = F_{tens} \cdot \sin ϴ$

$F_{tens} = F_y / \sin ϴ$
F_{tens} = (107.8 N)/sin(58.6)
F_{tens} = 126 N
(126 N in each cable)