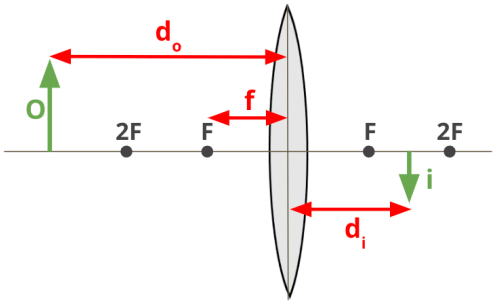
Mathematics of Lenses
Lesson Notes

Learning Outcomes
- How can the lens equation be used to solve Physics word problems?
- What is meant by magnification (M) and how can the M ratio be used in solving Physics word problems?

The Lens Equation
The mathematical relationship between object distance (d_o), image distance (d_i) and focal length (f) is given by the equation:

$$1/d_o + 1/d_i = 1/f$$



Sign Conventions for Variables d_o, d_i, and f
d_o is always a + value
d_i is + for real images and - for virtual images
f is + for converging lenses and - for diverging lenses

Problem Solving Strategy … Applied
Solving a lens equation problem requires careful reading, good conceptual reasoning, and an effective problem-solving strategy.

Example 1
Determine the image distance for a light bulb placed 45.0 cm from a converging lens having a focal length of 15.0 cm.

Given: $d_o = 45.0$ cm $f = +15.0$ cm

Unknown: $d_i = ???$

Formula: $1/d_o + 1/d_i = 1/f$

Algebra:
$1/(45.0) + 1/d_i = 1/(15.0)$
$1/d_i = 1/(15.0) - 1/(45.0) = 0.0444$
$d_i = 1/0.0444 \Rightarrow$ **$d_i = 22.5$ cm**

Effective Strategy
1. Read problem carefully.
2. ID given values; relate to variable symbols.
3. ID unknown variable.
4. ID the physics formula.
5. Substitute and solve algebraically.

Use the problem-solving strategy to solve Example 2. Show your solution.

Example 2
Determine the focal length of a lens that produces a virtual image that is 16.0 cm from the lens when the object is 28.5 cm from the lens.

Magnification

The **magnification (M)** of the image refers to how many times larger that the image is than the object: $M = h_i/h_o$
where h_i = image height and h_o refers to object height.

The ratio of heights equals the ratio of distances: $h_i/h_o = - d_i/d_o$

Sign Conventions for Variables d_o, d_i, h_o, h_i, and f

d_o is always a + value
h_o is always a + value
d_i is + for real images and - for virtual images
h_i is - for inverted (real) and + for upright (virtual) images
f is + for converging lens and - for diverging lenses.

Use the problem-solving strategy to solve Examples 3 and 4. Show your solution.

Example 3

A converging lens with a focal length of 32.0 cm produces a 6.2-cm tall, upright image when the object is 18.8 cm from the lens. Determine the object height and the image distance.

Example 4

The focal point is 22.5 cm from a diverging lens. A 5.0-cm tall light bulb is placed 48.1 cm from its surface. Determine the image distance and image height.