

Analytical Method of Vector Addition

Lesson Notes

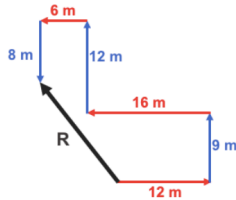
What is the Analytical Method?

Determining the resultant of two or more non-perpendicular vectors by adding all their x- and y-components.

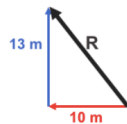
An Easy Problem Adding Perpendicular Vectors

Add the following vectors:

12 m, East
9 m, North
16 m, West
12 m, North
6 m, West
8 m, South



$\Sigma E-W = 10 \text{ m, West}$
 $\Sigma N-S = 13 \text{ m, North}$



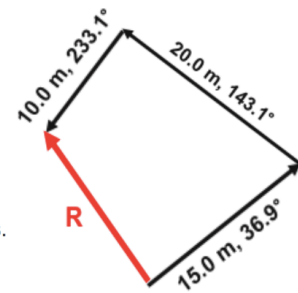
versus

A Difficult Problem Adding Non-Perpendicular Vectors

Add the following vectors:

15.0 m, 36.9°
20.0 m, 143.1°
10.0 m, 233.1°

To add non-perpendicular vectors, you must first resolve them into x- and y-components. Then add all the x- and y-components.



Simplifying a Difficult Problem to Make it an Easy Problem

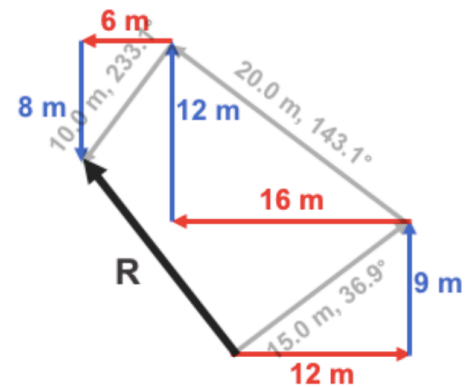
By resolving all non-perpendicular vectors into right angle components, a difficult problem can be transformed into an easier problem.

Replace 15.0 m, 36.9° with
12 m, East + 9 m, North

Replace 20.0 m, 143.1° with
16 m, West + 12 m, North

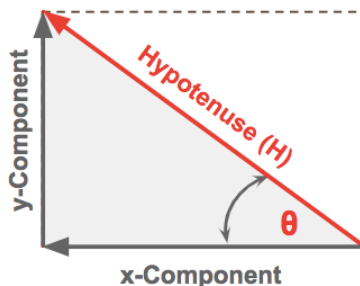
Replace 10.0 m, 233.1° with
6 m, West + 8 m, South

The **Resultant** is the same!



Trigonometric Method of Vector Resolution:

The **trigonometric method** of vector resolution relies on an understanding of the sine, cosine, and tangent functions.



SOH CAH TOA

Sin θ = **O**pposite/**H**ypotenuse

Cos θ = **A**djacent/**H**ypotenuse

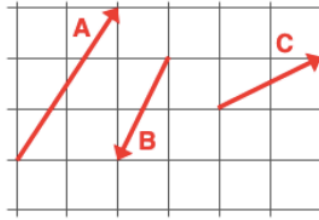
Tan θ = **O**pposite/**A**djacent

A Visual Example

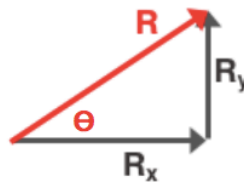
Add **A + B + C**

Scale:

Each square is 10 m
along its edge.



Vector	x	y
A	+20 m	+30 m
B	-10 m	-20 m
C	+20 m	+10 m
Resultant	+30 m	+20 m



$$R = \sqrt{(R_x^2 + R_y^2)}$$

$$R = \sqrt{(30^2 + 20^2)}$$

$$R = 36 \text{ m}$$

$$\theta = \tan^{-1}(R_y/R_x) = \tan^{-1}(20/30) = 34^\circ \text{ CCW}$$

Procedure for the Analytical Method of Vector Addition

Given 2 or more vectors to be added, use this procedure:

1. Sketch a vector addition diagram (as a quick estimate).
2. Create an x-y table; use trigonometric functions to resolve the given vectors into components.
3. Add all components to determine the components of the resultant (**R**). Sketch the resultant with **R_x** and **R_y** shown.
4. Use the Pythagorean theorem to determine the magnitude of the resultant (**R**).
5. Use a trigonometric function to determine the direction of the resultant (**R**).

Example 2

Use the 5-step method above to solve the following vector addition problem.

Add the following
vectors:

A: 4.50 km, 20.0°

B: 4.20 km, 270.0°

C: 6.00 km, 210.0°

Vector	x-Component	y-Component
A		
B		
C		
R		