## Energy on an Inclined Plane

One focus of study in physics is the concept of energy. There are two forms of mechanical energy that are of interest to physicists - kinetic energy and potential energy. Kinetic energy is the energy associated with an object's motion (i.e., its velocity). Potential energy is the energy stored in an object as a result of its vertical position (height). The total amount of mechanical energy is the sum of these two forms of energy.

A group of physics students are studying the relationship between the potential energy (PE), kinetic energy (KE) and total mechancial energy (TME) for a cart rolling up and down an incline. A computer-interfaced motion detector is placed at the top of the incline to detect changes in the height and the speed of the cart. The students start the detector, hold the cart for about 1.0 seconds and then give it a gentle push up the hill. The computer generates plots showing the position and height as a function of time. See Figure 1.

Figure 1



Figure 2: The Four Locations of the Cart (in cm)

The students analyze the graphs to determine the height above the table and the velocity at four different locations along the incline plane. See location labels (A, B, C and D) on Figure 2. The position values are in units of centimeters (cm). Location B is the highest point reached by the cart along the track. Using height and velocity values, the students calculate the KE, PE and TME for the

Table 1: Energy Values of Cart

| Location | KE (J) | PE (J) | TME (J) |
| :---: | :---: | :---: | :---: |
| A | 0.505 | 0.603 | 1.108 |
| B | 0.000 | 1.107 | 1.107 |
| C | 0.366 | 0.743 | 1.109 |
| D | 0.845 | 0.263 | 1.108 | four locations. See Table 1.

## Questions:

1. Which graph must the students use to determine the kinetic energy of the cart?
a. The position versus time graph must be used.
b. The velocity versus time graph must be used.
c. Both the position and the velocity versus time graphs must be used.
d. Neither graph is used. The kinetic energy values are calculated without using the graphs.
2. At which of these listed times does the cart have the lowest kinetic energy?
a. At 1.10 seconds.
b. At 2.25 seconds.
c. At 2.55 seconds.
d. At 3.12 seconds.
3. The formula for calculating the kinetic energy is $\mathbf{K E}=\mathbf{0 . 5} \cdot \mathrm{m}^{\bullet} \mathrm{v}^{2}$ where $\mathbf{m}$ represents the mass in kilograms and $\mathbf{v}$ represents the velocity in meter/second. How does the kinetic energy (KE) of the cart at 1.60 seconds compare to the kinetic energy (KE) of the cart at 1.92 seconds?
a. The KE at 1.60 seconds is about the same as the KE at 1.92 seconds.
b. The KE at 1.60 seconds is about one-half the value of the KE at 1.92 seconds.
c. The KE at 1.60 seconds is about two times greater than the KE at 1.92 seconds.
d. The KE at 1.60 seconds is about four times greater than the KE at 1.92 seconds.
e. The KE at 1.60 seconds is about one-fourth the value of the KE at 1.92 seconds.
4. For which of the following positions along the track does the cart have the greatest kinetic energy?
a. 30 cm
b. 50 cm
c. 90 cm
d. 140 cm
5. Which one of the following statements best describes the relationship between the cart's kinetic energy (KE), potential energy (PE) and total mechanical energy (TME).
a. As the PE increases, the KE and the TME are also observed to increase.
b. As the PE increases, the KE decreases, but the TME remains relatively constant.
c. The TME increases as the PE increases, but the KE remains relatively constant.
d. There seems to be no pattern in the manner in which the KE, PE and TME change.
6. Which combination of kinetic energy (KE), potential energy (PE) and total mechanical energy (TME) values would be expected when the cart is located at a position of 90 cm $(0.90 \mathrm{~m})$ from its starting location?
a. $\mathrm{KE}=0.43 \mathrm{~J}, \mathrm{PE}=0.68 \mathrm{~J}, \mathrm{TME}=1.11 \mathrm{~J}$
b. $\mathrm{KE}=1.80 \mathrm{~J}, \mathrm{PE}=1.63 \mathrm{~J}, \mathrm{TME}=3.43 \mathrm{~J}$
c. $\mathrm{KE}=1.63 \mathrm{~J}, \mathrm{PE}=1.80 \mathrm{~J}, \mathrm{TME}=3.43 \mathrm{~J}$
d. $\mathrm{KE}=0.26 \mathrm{~J}, \mathrm{PE}=0.85 \mathrm{~J}, \mathrm{TME}=1.11 \mathrm{~J}$

## Answers and Explanations

1. Answer: B

Explanation: As mentioned in the first paragraph of the passage, kinetic energy is the energy associated with an object's velocity. As such, the students must use the velocity versus time graph to determine the kinetic energy.
2. Answer: B

Explanation: As mentioned in the first paragraph of the passage, kinetic energy is the energy associated with an object's velocity. The kinetic energy will be least when the velocity is least. And so the time at which the velocity is least is the time at which the kinetic energy is least. So the strategy here is identify each listed time along the horizontal axis of the velocity-time graph (bottom graph) in Figure 1. Then trace vertically up to the point on the line at each time and horizontally across to the vertical axis in order to determine the velocity value. This velocity value is least for the time of 2.25 seconds.
3. Answer: D

Explanation: As indicated by the equation, the kinetic energy is greatest where the velocity value is the greatest. The velocity of the cart at the two indicated times can be read off the velocity-time graph (bottom graph) of Figure 1. At 1.60 seconds, the velocity of the cart is $+2.0 \mathrm{~m} / \mathrm{s}$. At 1.92 seconds, the velocity is $+1.0 \mathrm{~m} / \mathrm{s}$. Since the formula for kinetic energy shows that kinetic energy depends upon the square of the velocity, an object with twice the velocity will have four times the kinetic energy. Since the velocity at 1.60 seconds is two times the velocity at 1.92 seconds, the kinetic energy will be four times greater at the earlier time.
4. Answer: A

Explanation: The kinetic energy is dependent upon the velocity. Since velocity values are squared in the formula, it doesn't matter whether the velocity is a positive or a negative value. So the position for which the cart has the greatest absolute value of velocity is the position at which the kinetic energy is the greatest. This position and its corresponding velocity can be determined by using both the position-time graph (top graph) and the velocity-time graph (bottom graph) of Figure 1. For each indicated position, convert the cm value to a m value. For instance, the 30 cm is equivalent to 0.30 m . Then identify the position along the vertical axis of the position-time graph in Figure 1. Trace horizontally across from this position to the line on the graph. Once the point on the line at this position is determined, trace vertically downwards to the horizontal axis to determine the time. Once the time has been determined, use the time on the velocity-time graph of Figure 1 to determine the corresponding velocity. The absolute value of the velocity is greatest at a position of 30 cm (and 3.1 seconds). Of all listed positions, the kinetic energy is greatest at this one.
5. Answer: B

Explanation: The changes in potential energy and kinetic energy can be determined by analyzing the information in both graphs in Figure 1. Once the cart is given a push (ending around 1.45 seconds), it begins to roll up the hill. The upward motion occurs from about 1.45 seconds to 2.25 seconds. During this time, the position values are increasing. This
increase in position corresponds to a gain in height and thus in potential energy. During the same upward motion from 1.45 seconds until 2.25 seconds, the cart's velocity is decreasing from about $2.3 \mathrm{~m} / \mathrm{s}$ to a value of $0 \mathrm{~m} / \mathrm{s}$. This decrease in velocity is consistent with a decrease in kinetic energy. So as the potential energy increases, the kinetic energy is decreases. This same observation is confirmed by the data in rows 1-2 of Table 1. Furthermore, one observes from the lab column of Table 1, that as this increase in PE and decrease in KE occurs, the amount of TME remains relatively constant. And so choice B is the correct option to this question.
6. Answer: A

Explanation: The 90 cm position is cose to halfway in between the 80 cm position of location A and the 100 cm position of location C. By interpolation, one would expect the potential energy at this location to be approximately midway between the 0.603 J of position A and the 0.743 J of position C. Similarly, the KE will be approximately midway between the 0.505 J of location A and the 0.366 J of location C . The only option that meets this criteria is option A.

